Natural Tracking PID Process Control for Exponential Tracking

Ljubomir T. Grujić
Faculty of Mechanical Engineering, University of Belgrade, 11000 Belgrade, Serbia, Yugoslavia
William Pratt Mounfield, Jr.
Dept. of Mechanical Engineering, Louisiana State University, Baton Rouge, LA 70803

To cope with modeling uncertainties and randomness of external disturbances, a new tracking control called the natural control concept is designed. Its implementation is completely independent of the internal dynamics of a controlled system, its desired output and external disturbances. The design algorithm established ensures a prespecified exponential quality of output tracking. The theory presented in this article is applied effectively to the design of natural tracking control for a chemical reaction process described by the fourth-order, linear, state-space mathematical model.

Introduction

Chemical reactions and processes are characterized, from a control point of view, by the time variation of their outputs, uncertainties of their internal dynamics models, and randomness of external disturbances. From another control standpoint, they are characterized by their sharp requirements on accuracy and quality of their output tracking. All these restrictions impose a complex control task: the control should force a process to produce an output that tracks (follows) any accepted desired output under action of unpredictable disturbances and in spite of its internal dynamics uncertainty. Is this control task possible? Does the process enable the existence of such a control? To answer the first question, the second one has to be answered first. The clarification of the second question leads to a new qualitative system concept called *natural trackability*, which will be defined before the system description.

The concept of *natural trackability* will be defined differently from that of state controllability concept of Kalman (1961), the output controllability by Bertram and Sarachik (1960), the output functional controllability by Rosenbrock (1970), Wolovich (1974), Chen (1984), and the concept of the output reproducibility by Brocket and Mesarović (1965). Natural trackability will express a system property that enables the existence of a control defined without knowing the system internal dynamics to force the system to exhibit a prespecified tracking under action of any disturbance from a class of permitted disturbances. The necessary and sufficient conditions for the system natural trackability will be proved.

A desired high-quality tracking is elementwise exponential tracking. Such a prespecified tracking will be guaranteed by designing a control that can be implemented without using any information about the internal dynamics of the controlled system and external disturbances. The output error data and information about the implemented control itself will be the only information generating control action at the next instant.

The theory of the article further extends the tracking theory by Grujić (1987, 1988a,b, 1989).

System Description

A large class of chemical objects together with sensors and final controller devices (such as actuators with valves) can be described by:

$$\frac{dx}{dt} = Ax + Bu + Dd \tag{1a}$$

$$y = Cx + Ed. \tag{1b}$$

where

t = time
$x \in \mathbb{R}^n$ = state vector
$u \in \mathbb{R}^m$ = control vector
$d \in \mathbb{R}^p$ = disturbance vector
$y \in \mathbb{R}^m$ = output vector

The dimensions of the matrices A, B, C, D, and E are appropriate. For simplicity, x, u, and y will also denote the state vector function, the control vector function, and the output vector function, respectively. The matrices A, B, and E can be unknown; B and C are completely known and all state variables x_k, $k=1, \ldots, n$, can be unmeasurable, $x=(x_1 x_2 \ldots x_n)^T$. The motion of the system caused by an initial state x_o, disturbance action d and control action u over time interval $[0,t]$, which is considered at time t and corresponds to A is denoted by $\underline{x}(t;x_o;d,u;A)$, $\underline{x}(0;x_o;d,u;A) \equiv x_o$. Analogously, the real output response is $\underline{y}(t;y_o;d,u;A)$, $\underline{y}(0;y_o;d,u;A) \equiv y_o$, and the output error response is $\underline{e}(t;e_o;d,u,y_d;A) = y_d(t) - \underline{y}(t;y_o;d,u;A)$, where y_d is the desired output vector, $y_d \in \mathbb{R}^m$ at time t, e_0 is the initial error vector, and $e = y_d - y$ is the output error.

The class S_y of all the accepted desired outputs vector functions y_d comprises all defined, continuous, bounded and continuously differentiable y_d on $\mathbb{R}_+ = [0, +\infty)$ with bounded first derivative on $\mathbb{R}_+$. All permitted disturbance vector functions d compose S_d so that every $d \in S_d$ is defined, bounded and continuous on $\mathbb{R}_+$ in the case of $E=0$. Otherwise, every $d \in S_d$ is also continuously differentiable everywhere on $\mathbb{R}_+$ with bounded first derivative on $\mathbb{R}_+$.

Exponential Tracking

To clearly and simply define exponential tracking we shall make use of the next notation: $\|e\| = (e^T e)^{1/2}$, $e^k \equiv (e\, e^{(1)} \ldots e^{(k)})^T$ and I is the identity matrix of the appropriate order.

Definition 1

The system of Eq. 1 controlled by control u exhibits global kth-order exponential tracking over $\mathbb{R}^{n\times n} \times S_d \times S_y$, $[S_d \times S_y]$, if, and only if, for every $(A,d,y_d) \in \mathbb{R}^{n\times n} \times S_d \times S_y$, $[(d,y_d) \in S_d \times S_y]$, respectively, there exists positive real numbers α and β, $\alpha = \alpha(d,u,y_d;A) \geq 1$, $\beta = \beta(d,u,y_d;A)$, such that $\|\underline{e}^k(t;e_o^k;d,u,y_d;A)\| \leq \alpha \|e_o^k\| \exp(-\beta t)$ for every $(e_0,t) \in \mathbb{R}^m \times \mathbb{R}_+$.

By referring to this definition and the well-known properties of time-invariant linear systems, we can easily verify the following statement.

Lemma 1

If the law u is such that the output error obeys Eq. 2:

$$\sum_{i=0}^{\nu \geq 1} K_i \underline{e}^{(i)}(t;e_o;d,u,y_d;A) + K_I \int_0^t \underline{e}(\tau;e_0;d,u,y_d;A)d\tau = 0 \text{ for every}$$

$$(A,d,e_0,t,y_d) \in \mathbb{R}^{n\times n} \times S_d \times \mathbb{R}^m \times \mathbb{R}_+ \times S_y, [(d,e_0,t,y_d) \in S_d \times \mathbb{R}^m \times \mathbb{R}_+ \times S_y], \quad (2)$$

where K_i and K_I are such nonnegative diagonal matrices that $e = 0$ is asymptotically stable state of Eq. 2, then the system of Eq. 1 controlled by u exhibits global elementwise exponential tracking over $\mathbb{R}^{n\times n} \times S_d \times S_y$, $[S_d \times S_y]$.

Lemma 2

In case $\nu = 1$ and K_0, K_1 and K_I are such positive diagonal matrices that the matrix W (Eq. 3) is stable:

$$W = \begin{pmatrix} K_1^{-1} & 0 \\ 0 & K_1^{-1} \end{pmatrix} \begin{pmatrix} 0 & K_1 \\ -K_I & -K_0 \end{pmatrix}, \quad (3)$$

then the solution to Eq. 2 is found in the form of Eq. 4,

$$\underline{e}^1(t; e_0^1) = [\exp(Wt)]e_0^1, \quad (4)$$

so that it exponentially converges to $e^1 = 0$.

Proof

Let $\nu = 1$ and K_0, K_1 and K_I be such positive diagonal matrices that W (Eq. 3) is stable. After differentiating Eq. 2 we get $K_1 e^{(2)} + K_0 e^{(1)} + K_I e = 0$, or $e^{(2)} + K_1^{-1} K_0 e^{(1)} + K_1^{-1} K_I e = 0$ that can be set in the state space form:

$$\dot{e}^1 = \begin{pmatrix} \dot{e} \\ \dot{e}^{(1)} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -K_1^{-1}K_I & -K_1^{-1}K_0 \end{pmatrix} \begin{pmatrix} e \\ e^{(1)} \end{pmatrix} = \begin{pmatrix} K_1^{-1} & 0 \\ 0 & K_1^{-1} \end{pmatrix} \times \begin{pmatrix} 0 & K_1 \\ -K_I & -K_0 \end{pmatrix} \begin{pmatrix} e \\ e^{(1)} \end{pmatrix} = We^1.$$

Since the solution to this equation is given by Eq. 4, which is equivalent to Eq. 2, Eq. 4 determines the solution to Eq. 2. The solution of Eq. 4 exponentially converges to $e^1 = 0$ because the matrix W (Eq. 3) is stable. This completes the proof.

The matrices K_0, K_1 and K_I can be selected so that the solution $e^{(1)}$ (Eq. 4) possesses a prespecified tracking property and quality. Their choice is completely independent of the system of Eq. 1 and $(d,y_d) \in S_d \times S_y$.

Control u forcing the system of Eq. 1 to exhibit a prespecified tracking such as that determined by Eq. 4 for stable W (Eq. 3) will be called *tracking control*. If its implementation does not need knowledge of the matrix A and the state vector x, then it will be called *natural tracking control*. The adjective "natural" emphasizes its characteristic that is synthesized independent of the internal dynamics of the system of Eq. 1.

Natural Trackability

The question appears now whether for the system of Eq. 1 there exists a natural tracking control. In other words, what are the necessary and sufficient conditions for natural tracking control? This will be precisely explained via the next definition and theorem.

Definition 2

(i) The system of Eq. 1 is globally naturally trackable over $S_d \times S_y$, if, and only if, for any $T \in (0, +\infty)$ there is a control function u defined on $\mathbb{R}_+$ such that the control action on the system guarantees Eq. 5 for an unknown fixed $A \in \mathbb{R}^{n\times n}$,

$$\underline{e}(t;e_o;d,u,y_d;A) = 0 \text{ for all } t \in [T, +\infty) \text{ and any } (d,e_0,y_d) \in S_d \times \mathbb{R}^m \times S_y. \quad (5)$$

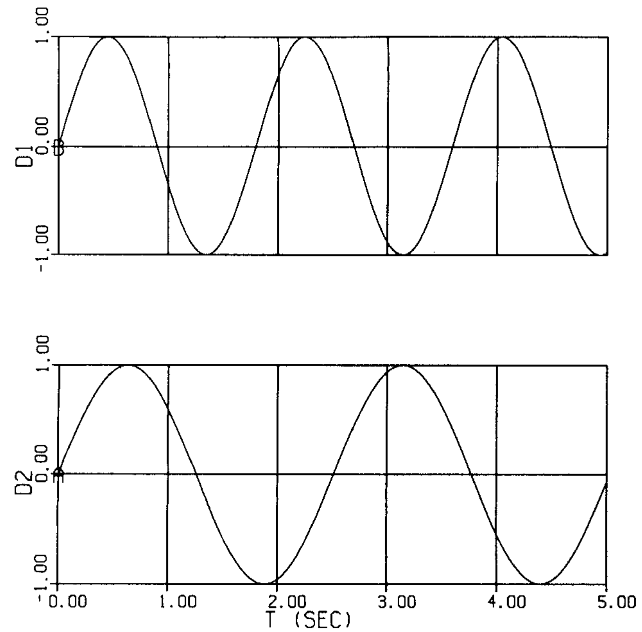

Figure 1. Disturbance inputs for all cases.

(ii) The system of Eq. 1 is globally naturally trackable over $\mathcal{R}^{n\times n}\times S_d\times S_y$, if, and only if, (i) holds for every $A\in\mathcal{R}^{n\times n}$.

The natural trackability links in some sense the output functional controllability (Wolovich, 1974; Chen, 1984; Rosenbrock, 1970) and disturbance rejection. However, since the internal dynamics of the system is unknown, conditions for natural trackability differ from those for output function controllability and for disturbance rejection. This is clarified as follows.

Theorem 1

For the system of Eq. 1 to be globally naturally trackable over $\mathcal{R}^{n\times n}\times S_d\times S_y$, it is both necessary and sufficient that

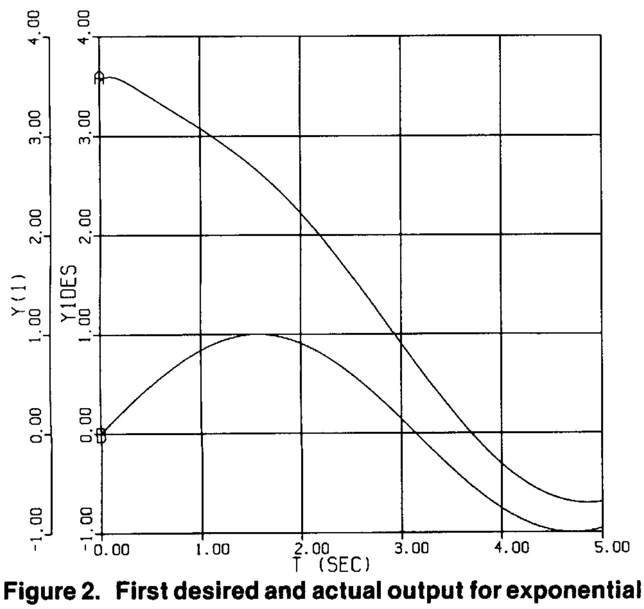

Figure 2. First desired and actual output for exponential tracking.

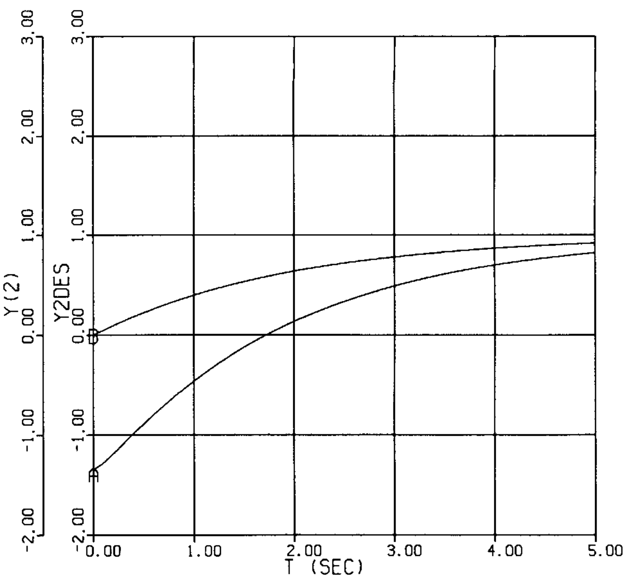

Figure 3. Second desired and actual output for exponential tracking.

$$\det(CB)\neq 0. \tag{6}$$

Proof

Necessity. Let the system of Eq. 1 be globally naturally trackable over $\mathcal{R}^{n\times n}\times S_d\times S_y$ and let $(A=0)\in\mathcal{R}^{n\times n}$ be accepted. Now, from Eqs. 1 and 5 we easily derive Eq. 7:

$$CB\int_0^t u(\tau)d\tau = y_d(t)-y_{do}+e_o-CD\int_0^t d(\tau)d\tau - E[d(t)-d_o],$$
$$\forall(d,e_o,t,y_d)\in S_d\times\mathcal{R}^m\times[T,+\infty)\times S_y. \tag{7}$$

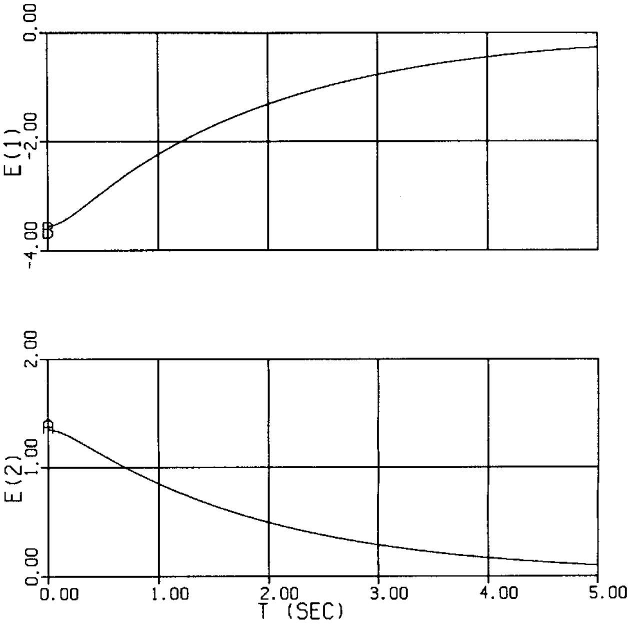

Figure 4. Tracking error between desired and actual outputs, exponential tracking.

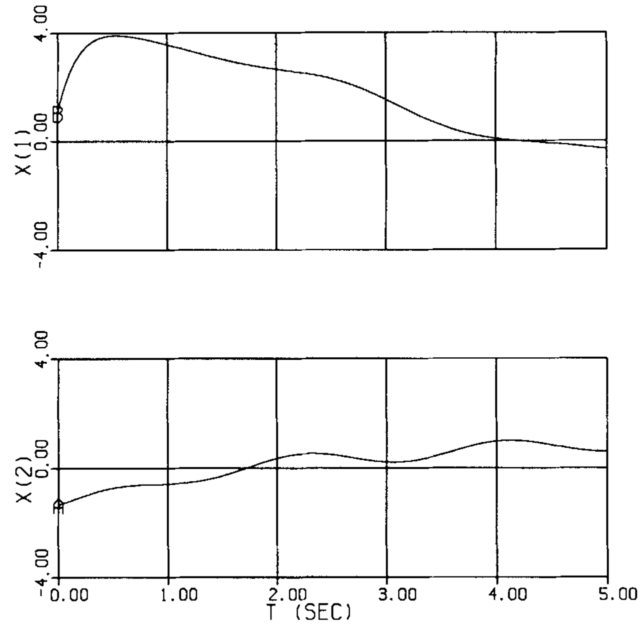

Figure 5. States X_1 and X_2 for exponential tracking.

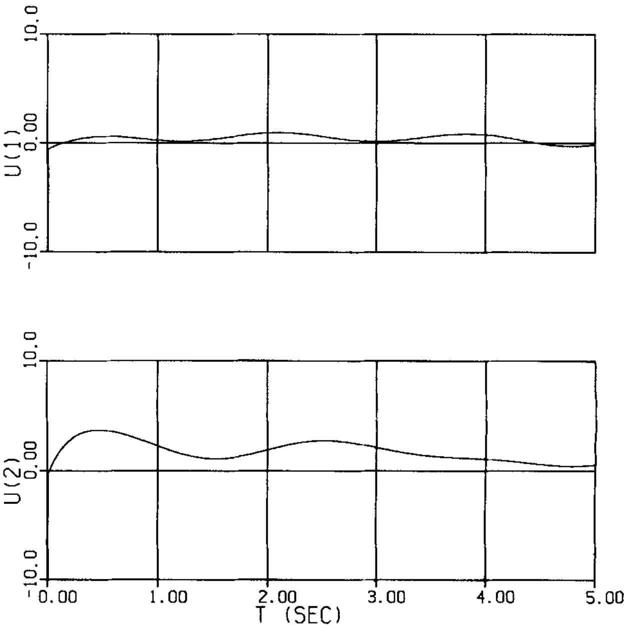

Figure 7. Natural tracking controls U_1 and U_2 for exponential tracking.

Since Eq. 7 is solvable in $\int_0^t u(\tau)d\tau$, if, and only if, det $(CB) \neq 0$, then Eq. 6 holds.

Sufficiency. Let $\epsilon \in [0^+, +\infty)$ and

$$u_f(t) = u(t-\epsilon). \tag{8}$$

From now on let $\epsilon = 0^+$ and $u(t^-)$ be $u(t-0^+)$ so that

$$u_f(t) = u(t-0^+) = u(t^-). \tag{9}$$

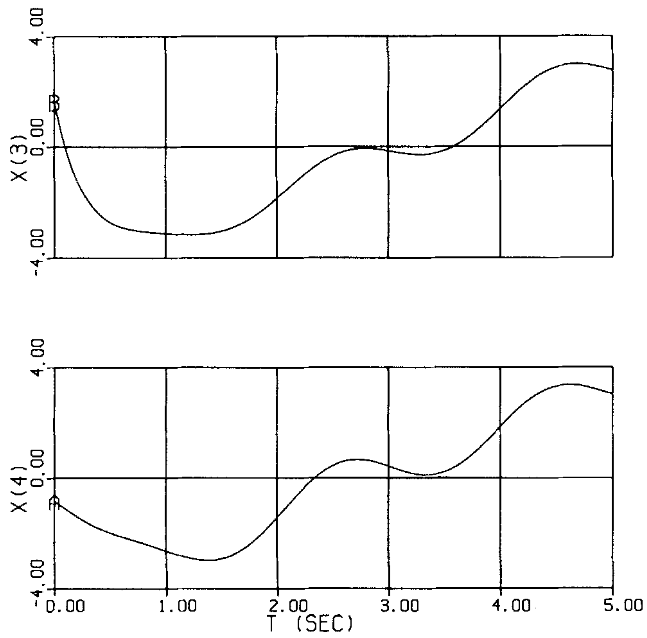

Figure 6. States X_3 and X_4 for exponential tracking.

Let for any $T \in (0, +\infty)$ a function φ: $\mathcal{R} \rightarrow \mathcal{R}^m$, $\varphi(t) \in C^{(1)}(\mathcal{R}_+)$, $\varphi(0) = -e(0)$, $\varphi(t) = 0$ for $t \in [T, +\infty)$, be arbitrarily chosen. Let for any $(A, d, e_o, y_d) \in \mathcal{R}^{n \times n} \times S_d \times \mathcal{R}^m \times S_y$ the control function u be defined by Eqs. 6, 9 and 10:

$$u(t) = u_f(t) + (CB)^{-1}[\underline{e}(t; e_o; d, u, y_d; A) + \varphi(t)] \text{ for all } t \in \mathcal{R}_+. \tag{10}$$

Since the closed-loop control system (Eqs. 1, 9 and 10) is linear and time-invariant, the smoothness property of every $(d, y_d) \in S_d \times S_y$ proves that $\underline{x}$ and $\underline{e}$ are continuously differentiable functions with bounded first derivatives so that:

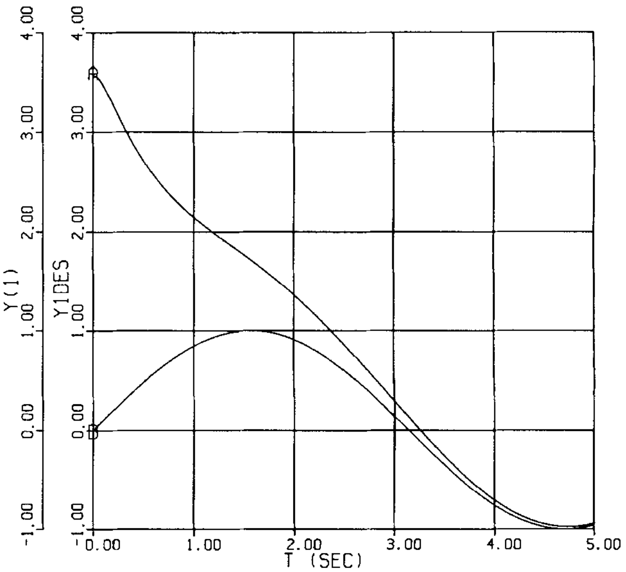

Figure 8. First desired and actual outputs for higher-quality tracking.

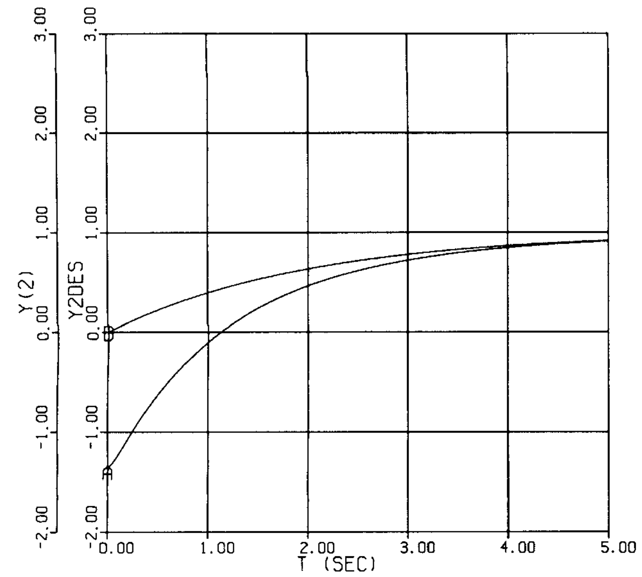

Figure 9. Second desired and actual output for higher-quality tracking.

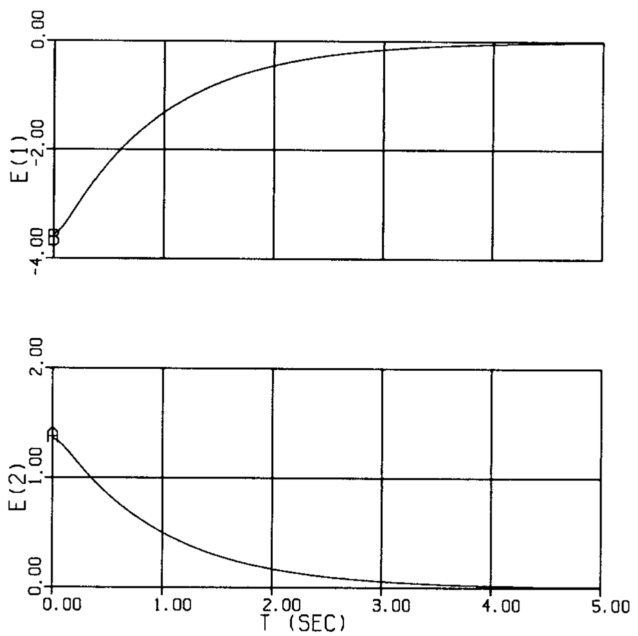

Figure 10. Tracking error E_1 and E_2 for higher-quality tracking.

$$x(t) \equiv x(t^-),\ x^{(1)}(t) \equiv x^{(1)}(t^-),\ e(t) \equiv e(t^-). \tag{11}$$

Now, Eqs. 1 and 9–11 imply:

$$(CB)^{-1}[\underline{e}(t;e_0;d,u,y_d;A) + \varphi(t)] = 0,\ \forall t \in \mathbb{R}_+. \tag{12}$$

The definition of φ and Eq. 12 yield $\underline{e}(t;e_0;d,u,y_d;A) = 0$, $\forall (A,d,e_0,t,y_d) \in \mathbb{R}^{n\times n} \times S_d \times \mathbb{R}^m \times [T, +\infty) \times S_y$. This result proves that the system of Eq. 1 controlled by control u (Eqs. 9 and 10), which is synthesized without using any information about the matrix A and the state vector $\underline{x}$, exhibits global tracking over $\mathbb{R}^{n\times n} \times S_d \times S_y$, that obeys Eq. 5.

Natural Tracking Control Synthesis

Let a tracking quality be defined by Eqs. 3 and 4, and let the control u be defined by Eq. 13:

$$u_f(t) = u(t^-) \tag{13a}$$

$$u(t) = u_f(t) + (CB)^{-1}[K_1 e^{(1)}(t) + K_0 e(t) + K_I \int_0^t e(\tau)d\tau],\ \forall t \in \mathbb{R}_+ \tag{13b}$$

Such a control does not depend on the internal dynamics of the controlled system (Eq. 1) or its state. It is defined only in terms of the output error data and the feedback control $u_f(t)$ representing the just realized control.

Theorem 2

For the system of Eq. 1 controlled by a natural tracking control u to exhibit global elementwise exponential tracking over $\mathbb{R}^{n\times n} \times S_d \times S_y$ defined by Eqs. 3 and 4 it is both necessary and sufficient that

(i) $\det(CB) \neq 0$,
(ii) the control u is defined by Eq. 13.

Proof

Necessity. Let the system of Eq. 1 controlled by u exhibit global, first-order exponential tracking over $\mathbb{R}^{n\times n} x S_d x S_y$ defined by Eqs. 3 and 4. Theorem 1 implies condition (i). From Eqs. 2 and 3, we obtain Eq. 14.

$$[K_1 e^{(1)}(t) + K_0 e(t) + K_I \int_0^t e(\tau)d\tau] = 0,\ \forall t \in \mathbb{R}_+. \tag{14}$$

This means also that $e(t)$ is continuously differentiable in $t \in \mathbb{R}_+$. Now, linearity of Eqs. 1 and 14, and smoothness of $(d,y_d) \in S_d \times S_y$ guarantee continuity of $x(t)$ and $x^{(1)}(t)$ in $t \in \mathbb{R}_+$:

$$x(t^-) \equiv x(t), x^{(1)}(t^-) \equiv x^{(1)}(t), \forall t \in \mathbb{R}_+. \tag{15a}$$

Hence,

$$x^{(1)}(t^-) = Ax(t^-) + Bu(t^-) + Dd(t^-) = Ax(t) + Bu(t^-) + Dd(t),\ \forall t \in \mathbb{R}_+, \tag{15b}$$

and

$$x^{(1)}(t) = Ax(t) + Bu(t) + Dd(t),\ \forall t \in \mathbb{R}_+. \tag{15c}$$

Now, Eqs. 14 and 15, together with Eq. 13a, prove Eq. 13b.

Sufficiency. Let conditions (i) and (ii) hold. Then, Eqs. 15 are true. They, together with Eq. 13, imply Eq. 14 which is equivalent to Eqs. 3 and 4. Hence, the statement of the theorem is correct.

Simulation Study

A linear, time-invariant, fourth-order, two-input, two-output natural tracking control system of an unstable batch process reaction was simulated to demonstrate first-order exponential tracking. The system dynamics were in the form of Eqs. 1 and 16 (Rosenbrock, 1974, p. 213):

$$A=\begin{bmatrix} -1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.24 & 0.0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}$$

$$B=\begin{bmatrix} 0.0 & 0.0 \\ 5.679 & 0.0 \\ 1.136 & -3.146 \\ 1.136 & 0.0 \end{bmatrix} \quad (16a)$$

$$C=\begin{bmatrix} 1.0 & 0.0 & 1.0 & -1.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \end{bmatrix}, D=\begin{bmatrix} 1.2 & -1.4 \\ 0.75 & 0.95 \\ 0.55 & -0.75 \\ 0.45 & -0.65 \end{bmatrix} \quad (16b)$$

$$x(0)=\begin{bmatrix} 1.05 \\ -1.35 \\ 1.65 \\ -0.85 \end{bmatrix}, E=\begin{bmatrix} 0.15 & -0.23 \\ 0.10 & 0.07 \end{bmatrix} \quad (16c)$$

The transfer matrix function for this system was:

$$G(s)=\frac{\begin{bmatrix} \dfrac{(29.2s+263.3)}{(5.679s^3+42.67s^2-68.84s-106.8)} & \dfrac{-(3.146s^3+32.67s^2+89.93s+31.81)}{(9.43s+15.15)} \end{bmatrix}}{(s^4+11.67s^3+15.76s^2-88.31s+5.514)}$$

Note that

$$CB=\begin{bmatrix} 0.0 & -3.146 \\ 5.679 & 0.0 \end{bmatrix}$$

for all these simulation runs which implies $\det(CB) \neq 0$ and proves global trackability of the system of Eq. 16 over $\mathbb{R}^{4\times 4} \times S_d \times S_y$.

The control algorithm used was:

$$u(t)=u(t^-)+(CB)^{-1}[K_1e^{(1)}(t)+K_0e(t)+K_I \int e(t)dt] \quad (17)$$

where initially $K_1=I$, $K_0=2I$, $K_I=I$. This specifies the tracking quality with a desired natural frequency of $\omega_n=1.0$ rad/s and damping ratio $\xi=1.0$. In this example,

$$(CB)^{-1}=\begin{bmatrix} 0.0 & 0.1761 \\ -0.3179 & 0.0 \end{bmatrix}.$$

The simulations were run with a low-pass filter on the desired output y_d to assure the inputs to the system complied with the conditions on continuity and differentiability. The filter was of the form:

$$\hat{y}_d(t)=\int_0^t k_f(y_d(\tau)-\hat{y}_d(\tau))d\tau+\hat{y}_d(0) \quad (18)$$

where k_f = 10,000. In Figures 1-13, the inputs y_d are plotted for reference, not the filtered versions to demonstrate the tracking quality of these systems with respect to the true desired output of the systems despite its filtering. The first derivative of the error $e^{(1)}(t)$ was generated by a first-order finite difference equation. All runs were simulated with a first-order integration method. The integration step size for all simulations remained at Δt = 0.000025 s.

The two disturbances $d(t)$ are bounded continuous functions of time, as shown in Figure 1 for $d_1(t)=\sin(3.5t)[D(1)]$ and $d_2(t)=\sin(2.5t)[D(2)]$. The natural controller used only the desired outputs and measured outputs to control $[y_1(t)]$ and $[y_2(t)]$. The output $y_1(t)$ $[Y(1)]$ in Figure 2 exponentially converges to the desired output $y_{1d}(t)=\sin(t)[YD(1)]$. The second output, $y_2(t)[Y(2)]$ in Figure 3 converges to the desired output $y_{2d}(t)=(1-e^{-0.5t})[YD(2)]$ in a similar manner. The exponential convergence of these two outputs to their respective desired outputs is demonstrated in Figure 4. The coupled, unstable system exhibits first-order exponential tracking as the error asymptotically converges to zero. In Figure 5, the states $x_1(t)[X(1)]$ and $x_2(t)[X(2)]$ are smooth and bounded; there is,

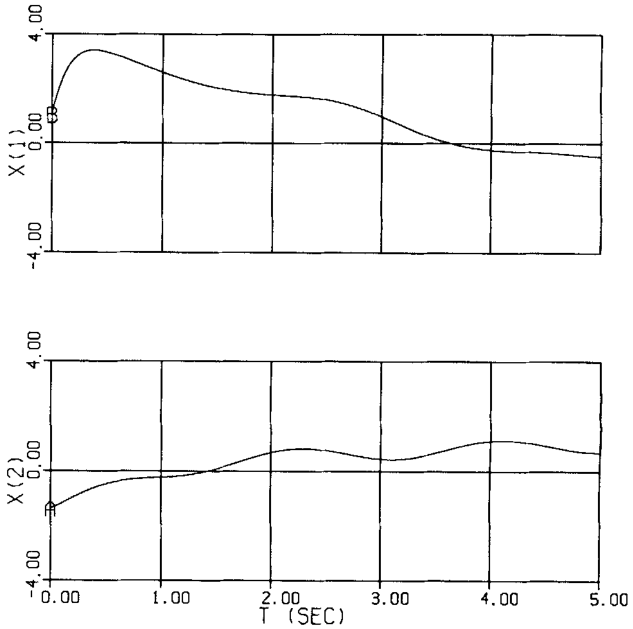

Figure 11. System states X_1 and X_2 under higher-quality tracking control.

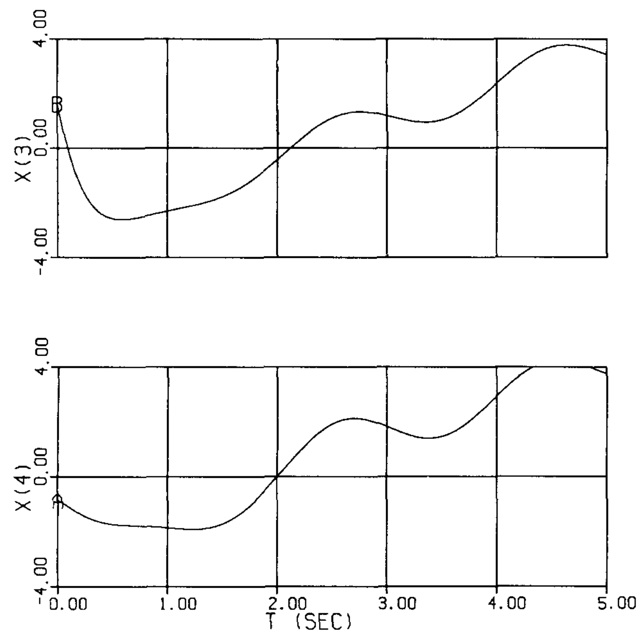

Figure 12. System states X_3 and X_4 under higher-quality tracking control.

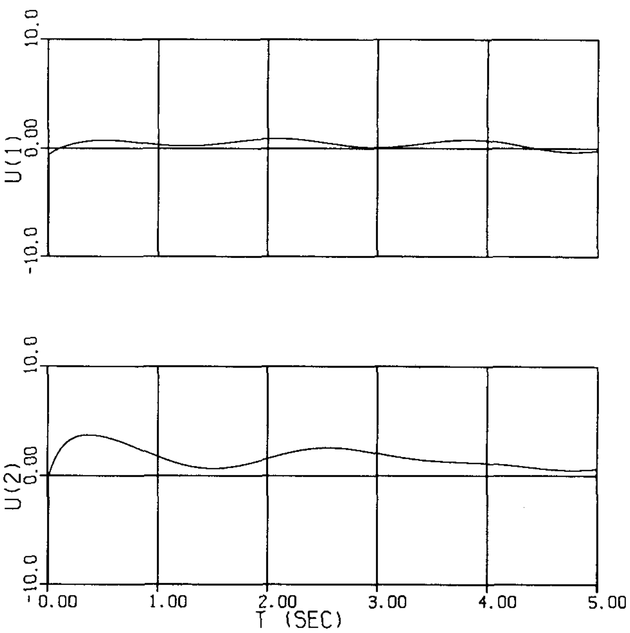

Figure 13. Natural tracking control for higher-quality tracking.

however, no control of the states, and only the outputs of this system are controlled. In Figure 6 the states $x_3(t)[X(3)]$ and $x_4(t)[X(4)]$ are also smooth and bounded. The control outputs $u_1(t)[U(1)]$ and $u_2(t)[U(2)]$ in Figure 7 show the result of the simultaneous compensation of the disturbance and forcing the outputs to obey the desired tracking exponential quality. There is no prior knowledge of the required control, that is $u(0)=0$.

The system tracking requirements were changed so that the desired natural frequency was $\omega_n = 2.0$ rad/s, while maintaining the damping ratio $\xi = 1.0$. The output $y_1(t)[Y(1)]$ in Figure 8 exponentially converges, twice as fast as before, to the desired output $y_{1d}(t) = \sin(2t)[YD(1)]$ from the same initial value with the same controller configuration but with gains $K_0 = 4I$, $K_I = 4I$ and $K_1 = I$. The second output, $y_2(t)[Y(2)]$ in Figure 9 converges to the desired output $y_{d2}(t) = (1 - e^{-0.5t})[YD(2)]$ in a similar manner with the faster (higher quality) tracking rate. The high-quality exponential convergence of these two outputs to their respective desired outputs is demonstrated in Figure 10. In Figure 11, the states $x_1(t)$ $[X(1)]$ and $x_2(t)[X(2)]$ remain smooth and bounded. In Figure 12, the states $x_3(t)[X(3)]$ and $x_4(t)[X(4)]$ also remain smooth and bounded. The control outputs $u_1(t)[U(1)]$ and $u_2(t)[U(2)]$ in Figure 13 show the result of the simultaneous compensation of the disturbance and forcing the outputs to obey the desired higher-quality exponential tracking.

Conclusion

An important control feature of a system is discovered and defined as its *natural trackability*. The necessary and sufficient condition for it is proved and used to synthesize a control without using any information about the system internal dynamics and state. Such a *natural tracking control* guarantees a prespecified high-quality exponential tracking in spite of actions of unmeasured external disturbances.

The demonstration of natural control used linearly-independent inputs as the desired outputs. The tracking quality was prespecified by the choice of gains on the PID controller. The system outputs exponentially converge to the respective linearly-independent desired outputs. These outputs are unaffected by the measured, unmodeled disturbances. Each of the two output errors decays exponentially with this prespecified rate of convergence. The state variables and control variables were all the time within narrow bounds. And finally, the controller was designed with no knowledge of the internal dynamics of the system to be controlled.

Notation

$A \in \mathcal{R}^{n\times n}$ = matrix describing the internal dynamics of a system, which can be completely unknown
$B \in \mathcal{R}^{n\times m}$ = matrix expressing characteristics of the system-controlling components such as controlling valves, B is completely known
$C \in \mathcal{R}^{m\times n}$ = matrix expressing characteristics of measuring devices (sensors), C is fully known
$D \in \mathcal{R}^{n\times p}$ = matrix describing the transmission of external disturbance actions on the system dynamics, D can be completely unknown
$d(.): \mathcal{R} \to \mathfrak{F}^p$ = vector function of external disturbances
$d \in \mathcal{R}^p$ = vector of external disturbances
$I \in \mathcal{R}^{m\times m}$ = identity matrix
$E \in \mathcal{R}^{m\times p}$ = matrix describing the transmission of disturbance directly on the system output, E can be completely unknown
$e(.): \mathcal{R} \to \mathfrak{F}^m$ = system output-error (vector) function, $e(t) = y_d(t) - y(t)$
$\underline{e}(.): \mathcal{R} \times \mathcal{R}^m \times \mathfrak{F}^p \times \mathfrak{F}^m \times \mathfrak{F}^m \times \mathcal{R}^{n\times n} \to \mathfrak{F}^m$ = system output error (vector) response $\underline{e}[t; e_0; d(.), u(.), y_d(.); A] = y_d(.) - \underline{y}[t; y_0; d(.), u(.); A]$
$e \in \mathcal{R}^m$ = output error vector, $e = (e_1 e_2 \ldots e_m)^T$
$\|e\|$ = $(e^T e)^{1/2}$
$e^k \in \mathcal{R}^{(k+1)m}$ = matrix of k-order derivatives of the output error, $i = 0,1,\ldots$
$\mathfrak{F}^k$ = k-dimensional functional space

$K_i \in [0,+\infty)$ = nonnegative diagonal matrices for controller's k-order derivatives, $k=1,2,...$
$K_I \in [0,+\infty)$ = nonnegative diagonal matrices for controller's integrator constants
$S_d \subset \mathcal{F}^p$ = set of all permitted $d(.)$
$S_y \subset \mathcal{F}^m$ = set of all accepted realizable $y_d(.)$
$\mathcal{R}_T \subset \mathcal{R}$ = time interval: $\mathcal{R}_T = [0,T)$, $T \in (0,+\infty)$
$t \in \mathcal{R}$ = time, $t_0 = 0$ is the initial time
$T \in (0,+\infty)$ = time T after which $e = 0$
$u(.):\mathcal{R} \to \mathcal{F}^m$ = control (vector) function
$u \in \mathcal{R}^m$ = control vector
$u(t-\epsilon)|_{\epsilon=0+} = u(t^-)$ = control vector of the just realized control at time t
$W \in \mathcal{R}$ = prespecified convergence rate for the k-order derivative of the output error
$x \in \mathcal{R}^n$ = system state vector
$\underline{x}(.):\mathcal{R} \times \mathcal{R}^n x \mathcal{F}^p \times \mathcal{F}^m \to \mathcal{F}^n$ = system motion, $\underline{x}[t; x_0; d(.), u(.)] \equiv x(t)$, $\underline{x}[0; x_0; d(0), u(0)] \equiv x_0$
$y(.):\mathcal{R} \to \mathcal{R}^m$ = system output (vector) function
$y \in \mathcal{R}^m$ = system output vector
$\underline{y}(.):\mathcal{R} \times \mathcal{R}^m \times \mathcal{F}^p \times \mathcal{F}^m \times \mathcal{R}^{n \times n} \to \mathcal{F}^m$ = system output (vector) response, $\underline{y}[t; y_0; d(.), u(.); A] \equiv y(t)$, $\underline{y}[0; y_0; d(0), u(0); A] \equiv y_0$
$y_d(.):\mathcal{R} \to \mathcal{F}^m$ = desired output response of the system
$y_d \in \mathcal{R}^m$ = desired output vector

Greek letters

$\alpha \in \mathcal{R}_+$ = positive scaling factor for the envelope of maximum exponential convergence of the output error
$\beta \in \mathcal{R}_+$ = maximum exponential convergence rate of the output error
$\epsilon \in [0,+\infty)$ = $\epsilon = 0^+$, time delay in the local controller's positive feedback
$\tau \in \mathcal{R}_+$ = integration variable
$\varphi \in \mathcal{R} \to \mathcal{R}^n$ = vector function of the initial output error

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